

Aging to non-Newtonian hydrodynamics in a granular gas

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Abstract. - The evolution to the steady state of a granular gas subject to simple shear flow is analyzed by means of computer simulations. It is found that, regardless of its initial preparation, the system reaches (after a transient period lasting a few collisions per particle) a non-Newtonian (unsteady) hydrodynamic regime, even at strong dissipation and for states where the time scale associated with inelastic cooling is shorter than the one associated with the irreversible fluxes. Comparison with a simplified rheological model shows a good agreement.

Introduction. – A granular gas is a large collection of (mesoscopic or macroscopic) particles which collide inelastically and are usually kept in a state of continuous agitation. Apart from their interest in industrial and technological applications, granular gases are important at a fundamental level as physical systems intrinsically out of equilibrium and thus exhibiting a wide spectrum of complex behavior [1–6]. Although the number of grains in a fluidized granular system is of course much smaller than the number of atoms or molecules in a conventional gas, it is large enough as to make nonequilibrium statistical-mechanical concepts and tools applicable. In particular, a kinetic theory approach (based on the Boltzmann and Enskog equations suitably modified to account for inelastic collisions) has proven to be very useful [6]. However, because of the energy dissipation upon collisions and the associated lack of detailed balance and Gibbs equilibrium state, one is not allowed to take for granted any phenomenology that applies to normal gases, unless much caution is exercised. Quoting Kadanoff, “one might even say that the study of granular materials gives one a chance to reinvent statistical mechanics in a new context” [1].

Hydrodynamics is one of the key features of standard fluids. As is well known, the hydrodynamic description of a conventional fluid consists of closing the exact balance equations for the densities of the conserved quantities (mass, momentum and energy) with constitutive equations relating the momentum and heat fluxes to the conserved densities (usually referred to as hydrodynamic fields) and their gradients. If the hydrodynamic gradients are weak, the fluxes can be assumed to be linear in those gradients, what results in the Navier–Stokes (NS) hydrodynamic description. On the other hand, even if the gradients are strong, a (non-Newtonian) hydrodynamic regime beyond the NS one is still possible. The conventional

scenario for the “aging to hydrodynamics” in a normal gas can be summarized as follows [7]. Given an arbitrary initial state, the evolution proceeds along two successive stages. First, during the so-called *kinetic* stage there is a fast relaxation (lasting a few collision times) to a “universal” (or “normal”) velocity distribution $f(\mathbf{r}, \mathbf{v}; t)$ that is a functional of the hydrodynamic fields (number density, flow velocity and temperature). Subsequently, the *hydrodynamic* stage is described through a slower evolution of the hydrodynamic fields as they approach equilibrium or an externally imposed nonequilibrium steady state. The first stage is sensitive to the initial preparation of the system, while in the hydrodynamic regime the system has practically “forgotten” the details of its initial state (except for an implicit dependence on the initial conditions through the hydrodynamic fields). If the hydrodynamic gradients are small enough when the hydrodynamic state is reached, the latter can be described by the NS terms in the Chapman–Enskog expansion [8]. However, a normal (or hydrodynamic) velocity distribution function is not restricted to the NS domain but can apply to the non-Newtonian regime as well [9].

The applicability of a hydrodynamic description to granular fluids is not self-evident at all [1]. In particular, the absence of energy conservation gives rise to a sink term in the energy balance equation which might preclude, except perhaps in quasi-elastic situations, the role of the (granular) temperature as a hydrodynamic variable. At least in the NS regime, however, there exists compelling evidence from theory [10], simulations [11] and experiments [12] supporting the validity of a hydrodynamic treatment of granular fluids. On the other hand, the inherent lack of scale separation in sheared granular gases invalidates a NS description, so that the applicability of hydrodynamics in those cases has been controversial [13].

The scenario of aging to hydrodynamics described above is known to hold in two limiting cases: finite dissipation but no gradients (*i.e.*, in the approach to the homogeneous cooling state) [14] and zero dissipation but large gradients [9]. Having that in mind, the question we want to address in this paper is, does that scenario still apply to granular gases with *strong* dissipation in the *non-Newtonian* regime?

Uniform shear flow. — In order to contribute to an understanding of the previous question and its possible answer, we will focus on inelastic hard spheres in the so-called simple or uniform shear flow (USF), which is characterized by a constant density n , a time-dependent (but uniform) granular temperature $T(t)$ and a flow velocity with a linear profile $\mathbf{u}(\mathbf{r}) = ay\hat{\mathbf{x}}$, a being the constant shear rate. There are three basic reasons to choose this state to isolate the problem at hand. First, it is macroscopically simple since only a hydrodynamic gradient exists ($a = \partial u_x / \partial y$) and moreover it is a constant. Second, this flow possesses a steady state (resulting from the balance between inelastic cooling and viscous heating) that is inherently non-Newtonian [13, 15]. Finally, since both the density n and the flow velocity $\mathbf{u}(\mathbf{r})$ are independent of time, the possible aging to hydrodynamics is enslaved by the temporal evolution of the granular temperature, which is precisely the quantity casting doubts on the applicability of hydrodynamics.

In the USF the mass continuity equation is identically satisfied, while momentum conservation implies that the shear stress P_{xy} is uniform. The energy balance equation becomes

$$\partial_t T(t) = -(2a/3n)P_{xy}(t) - \zeta(t)T(t), \quad (1)$$

where $\zeta(t)$ is the cooling rate due to inelasticity. Given a shear rate a and a coefficient of normal restitution α , a stationary temperature $T_s = \lim_{t \rightarrow \infty} T(t)$ is reached when the viscous heating and the inelastic cooling cancel each other. This steady state can be conveniently described by introducing dimensionless quantities. Let $\nu(T) = nT/\eta_0(T) \propto nT^{1/2}$, where $\eta_0(T)$ is the NS shear viscosity in the elastic case, denote an effective collision frequency. We can then define a reduced shear rate $a^*(t) = a/\nu(T(t))$, a reduced cooling rate $\zeta^*(t) = \zeta(t)/\nu(T(t))$ and a reduced shear stress $P_{xy}^*(t) = P_{xy}(t)/nT(t)$. Thus, one has $\frac{2}{3}a_s^*|P_{xy,s}^*| = \zeta_s^*$ in the steady state. The stationary temperature is approached from below ($T(t) < T_s$) if the shear rate is large enough and/or the inelasticity is small enough so that the heating rate (due to viscous effects) $\frac{2}{3}a^*(t)|P_{xy}^*(t)|$ prevails over the cooling rate (due to inelasticity) $\zeta^*(t)$; we will refer to this situation as the *heating* case. Otherwise, the stationary temperature is reached from above and this will be referred to as the *cooling* case. In this context, the question posed above can be rephrased as, does an *unsteady* hydrodynamic regime exist in both the heating and cooling cases *before* the steady state is achieved?

Restricting ourselves to low density granular gases and to states that are uniform in the Lagrangian frame of reference, so that $f(\mathbf{r}, \mathbf{v}; t) = f(\mathbf{V}(\mathbf{r}), t)$ with the peculiar velocity $\mathbf{V}(\mathbf{r}) = \mathbf{v} - \mathbf{u}(\mathbf{r})$, the Boltzmann equation becomes [9, 15]

$$(\partial_t - aV_y \partial_{V_x}) f(\mathbf{V}, t) = J[\mathbf{V}|f(t)], \quad (2)$$

where $J[\mathbf{V}|f(t)]$ is the (inelastic) Boltzmann collision operator. Equation (2) must be complemented with an initial condition $f(\mathbf{V}, 0) = f^0(\mathbf{V})$, so that the solution is actually a functional of the initial distribution, *i.e.*, $f(\mathbf{V}, t) = f(\mathbf{V}, t|f^0)$. Analogously, the velocity moments of f , such as the pressure tensor $P_{ij}(t|f^0)$ are also functionals of f^0 . Since the only time-dependent hydrodynamic variable is the temperature $T(t)$ and the only hydrodynamic gradient is the shear rate a , the existence of a hydrodynamic regime implies that, after a certain number of collisions per particle,

$$f(\mathbf{V}, t|f^0) \rightarrow n[m/2T(t)]^{3/2} f^*(\mathbf{C}(t); a^*(t)), \quad (3)$$

where $\mathbf{C}(t) \equiv \mathbf{V}/\sqrt{2T(t)/m}$ is the (peculiar) velocity in units of the (time-dependent) thermal speed, m being the mass of a grain. The scaled velocity distribution function $f^*(\mathbf{C}; a^*)$ is, for a given value of the coefficient of restitution α , a *universal* function in the sense that it is independent of the initial state f^0 and depends on the applied shear rate a through the reduced scaled quantity a^* only. In other words, if a hydrodynamic description is possible, the different solutions $f(\mathbf{V}, t|f^0)$ of the Boltzmann equation (2) would be “attracted” by the universal form (3). For asymptotically long times, the steady state would eventually be reached, *i.e.*, $f^*(\mathbf{C}; a^*) \rightarrow f_s^*(\mathbf{C}) = f^*(\mathbf{C}; a_s^*)$. Equation (3) has its counterpart at the level of the velocity moments. In particular, the pressure tensor $P_{ij}(t|f^0)$ would become

$$P_{ij}(t|f^0) \rightarrow nT(t)P_{ij}^*(a^*(t)) \quad (4)$$

with universal functions $P_{ij}^*(a^*)$. The non-Newtonian character of the hydrodynamic regime can be characterized by the nonlinear (reduced) shear viscosity $\eta^*(a^*)$ and viscometric function $\Psi^*(a^*)$ defined by

$$\eta^*(a^*) = -\frac{P_{xy}^*(a^*)}{a^*}, \quad \Psi^*(a^*) = \frac{P_{xx}^*(a^*) - P_{yy}^*(a^*)}{a^{*2}}. \quad (5)$$

Rheological model. — Before presenting the simulation results, it is worth considering a simple kinetic model [16] in which the Boltzmann collision operator $J[\mathbf{V}|f(t)]$ is replaced by a relaxation-time term toward the local equilibrium distribution plus a drag-force term mimicking the cooling effects due to inelastic collisions. Applied to the USF, it closes eq. (1) with the evolution equations [15, 17, 18]:

$$\partial_t P_{xy} = -aP_{yy} - \beta\nu(T)P_{xy} - \zeta(T)P_{xy}, \quad (6)$$

$$\partial_t P_{yy} = -\beta\nu(T)(P_{yy} - nT) - \zeta(T)P_{yy}, \quad (7)$$

where $\zeta(T) = \zeta^* \nu(T)$ with $\zeta^* = \frac{5}{12}(1 - \alpha^2)$ and here we take $\beta = \frac{1}{2}(1 + \alpha)$. Equations (6) and (7) can also be obtained, with $\beta = \frac{1}{6}(1 + \alpha)(2 + \alpha)$, from the Boltzmann equation in Grad's approximation [15, 19].

The set of coupled equations (1), (6) and (7) must be solved numerically [15], the solution including both the transient kinetic regime and the hydrodynamic evolution stage. In order to extract the hydrodynamic solution in an *analytical* way, we will introduce an additional simplification similar to that carried out in the elastic case [9, 20]. As mentioned before, $\nu(T) \propto nT^q$ with $q = \frac{1}{2}$. However, here we will temporarily view q as a (small) free parameter, so that the solutions to eqs. (1), (6) and (7) depend parametrically on q [21]. If $q = 0$, then $\nu = \text{const}$ and $a^* = \text{const}$, so there is no steady state (except if a^* takes a specific value). However, for sufficiently long times the scaled quantities P_{ij}^* reach well defined (hydrodynamic) values which depend on a^* and not on the initial state [22]. Next, carrying out a first-order perturbation analysis around $q = 0$ and constructing Padé approximants, one finally gets [22]

$$\eta^*(a^*) = \beta^{-1} [1 + 2\gamma(a^*)]^{-2} \left\{ 1 + \frac{1}{2} [\zeta^*/\beta - 2\gamma(a^*)] \times [1 - 6\gamma(a^*)] / [1 + 6\gamma(a^*)]^2 \right\}^{-1}, \quad (8)$$

$$\Psi^*(a^*) = 2\beta^{-2} [1 + 2\gamma(a^*)]^{-3} \left\{ 1 + \frac{\zeta^*/\beta - 2\gamma(a^*)}{2[1 + 6\gamma(a^*)]^2} \right\}^{-1} \times \left\{ 1 + \frac{\zeta^*/\beta - 2\gamma(a^*)}{[1 + 6\gamma(a^*)]^2} \right\}^{-1}, \quad (9)$$

where $\gamma(a^*) \equiv \frac{2}{3} \sinh^2 \left[\frac{1}{6} \cosh^{-1}(1 + 9a^{*2}/\beta^2) \right]$ and we have already set $q = \frac{1}{2}$. Equations (8) and (9) provide the hydrodynamic non-Newtonian viscosity and viscometric function as explicit functions of the reduced shear rate a^* (and of the coefficient of restitution α) in our simplified rheological model. The steady state corresponds to just one point on the curves $\eta^*(a^*)$ and $\Psi^*(a^*)$. In the model, it is given by $a_s^* = \sqrt{3\zeta^*/2\beta(\beta + \zeta^*)}$ [15, 17, 18], which verifies the condition $\gamma(a_s^*) = \zeta^*/2\beta$, and so $\eta_s^* = \eta^*(a_s^*) = \beta/(\beta + \zeta^*)^2$ and $\Psi_s^* = \Psi^*(a_s^*) = 2\beta/(\beta + \zeta^*)^3$. If we formally take the limit of vanishing shear rate, we recover the NS viscosity of the inelastic gas predicted by the model [18], namely $\lim_{a^* \rightarrow 0} \eta^*(a^*) = (\beta + \zeta^*/2)^{-1}$, as well as the Burnett value $\lim_{a^* \rightarrow 0} \Psi^*(a^*) = 2/(\beta + \zeta^*/2)(\beta + \zeta^*)$. We have checked that, for $\alpha \geq 0.5$, Eqs. (8) and (9) yield values which are hardly distinguishable from those obtained by a numerical solution of the set (1), (6) and (7) [22].

Monte Carlo simulations. — In order to test whether the hydrodynamic regime (3) exists or not, we have solved the Boltzmann equation (2) by means of the direct simulation Monte Carlo (DSMC) method [23]. As in ref. [24], we have taken $N = 10^4$ simulated particles and a varying time step $\delta t = 10^{-3} \tau^0 \sqrt{T^0/T}$, where T^0 is the initial temperature and $\tau^0 = \lambda/\sqrt{2T^0/m} \simeq 0.8885\nu^{-1}(T^0)$

is the initial mean free time, λ being the mean free path. In order to improve the statistics, the results are averaged over 100 independent realizations. The values of the coefficient of restitution considered have been $\alpha = 0.5, 0.7$ and 0.9 . For each value of α , four different shear rates have been taken: $a = 0.01/\tau^0$, $a = 0.1/\tau^0$, $a = 4/\tau^0$ and $a = 10/\tau^0$. The two first values of the shear rate ($a = 0.01/\tau^0$ and $a = 0.1/\tau^0$) are small enough to correspond to cooling cases, even for the least inelastic system ($\alpha = 0.9$), while the other two values ($a = 4/\tau^0$ and $a = 10/\tau^0$) are large enough to correspond to heating cases, even for the most inelastic system ($\alpha = 0.5$). For each one of the twelve pairs (α, a) , five widely different initial conditions have been chosen, so that sixty independent states have been simulated. The first initial condition (here referred to as A) is just the (local) equilibrium one, namely

$$f^0(\mathbf{V}) = n (m/2\pi T^0)^{3/2} e^{-mV^2/2T^0}. \quad (10)$$

The other four initial conditions are of the anisotropic form

$$f^0(\mathbf{V}) = (n/2) (m/2\pi T^0)^{1/2} e^{-mV_z^2/2T^0} \times [\delta(V_x - V^0 \cos \phi) \delta(V_y + V^0 \sin \phi) + \delta(V_x + V^0 \cos \phi) \delta(V_y - V^0 \sin \phi)] \quad (11)$$

where $\delta(x)$ is Dirac's distribution, $V^0 \equiv \sqrt{2T^0/m}$ is the initial thermal speed and $\phi \in [0, \pi]$ is an angle characterizing each specific condition. The pressure tensor corresponding to eq. (11) is given by $P_{xx}^0 = 2nT^0 \cos^2 \phi$, $P_{yy}^0 = 2nT^0 \sin^2 \phi$, $P_{zz}^0 = nT^0$ and $P_{xy}^0 = -nT^0 \sin 2\phi$. The four values of ϕ considered are $\phi = k\pi/4$ with $k = 0, 1, 2$ and 3 ; we will denote the respective initial conditions of type (11) as B0, B1, B2 and B3. Note that $P_{xy}^0 = 0$ in the initial conditions A, B0 and B2, while $P_{xy}^0 = -nT^0$ in the case of B1. On the other hand, P_{xy}^0 takes an “artificial” positive value ($P_{xy}^0 = nT^0$) if the system is initially prepared with the condition B3. According to eq. (1), this implies that during the very first times the fluid undergoes a double cooling effect: the inelastic cooling plus an artificial shear stress cooling. It takes some time before a “natural” negative value of $P_{xy}(t)$ is established and the usual viscous heating effect competes against the inelastic cooling.

Results and discussion. — The solution of the Boltzmann equation (2) through the DSMC method allows one to monitor the temporal evolution of velocity moments, such as the temperature $T(t)$ and the pressure tensor $P_{ij}(t)$, as well as of the velocity distribution function $f(\mathbf{V}, t)$ itself. In particular, one can follow the reduced shear rate $a^*(t) = a/\nu(T(t))$ (which decreases in the heating states and increases in the cooling states) and the reduced shear viscosity $\eta^*(t) = -P_{xy}(t)/nT(t)a^*(t)$. A parametric plot of $\eta^*(t)$ vs $a^*(t)$ is then useful to test the establishment of a hydrodynamic regime in which

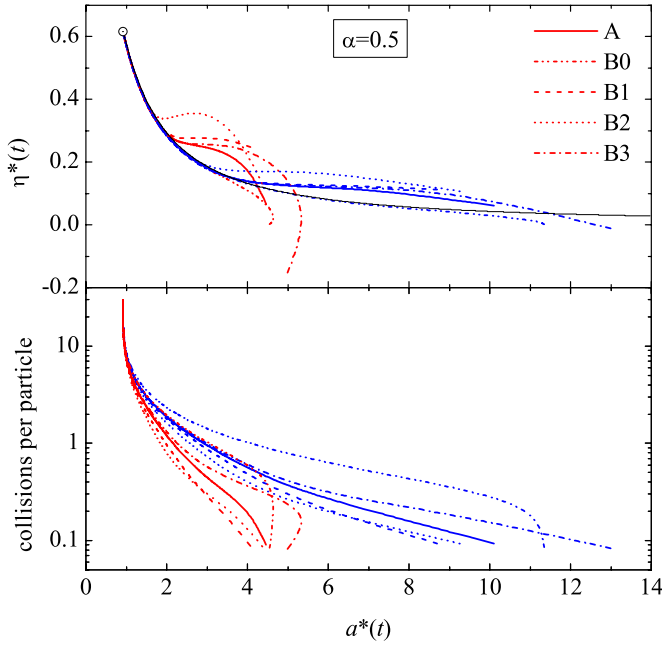


Fig. 1: (Color online) Reduced shear viscosity $\eta^*(t)$ (top panel) and number of collisions per particle (bottom panel) versus the reduced shear rate $a^*(t)$ for $\alpha = 0.5$ in the two heating cases $a = 4/\tau^0$ (red lines) and $a = 10/\tau^0$ (blue lines). In the top panel, the circle represents the steady-state point (a_s^*, η_s^*) , while the thin solid line corresponds to the hydrodynamic function, eq. (8), obtained from our simplified rheological model. Note the logarithmic scale in the vertical axis of the bottom panel.

$\eta^*(t) \rightarrow \eta^*(a^*(t))$, where the function $\eta^*(a^*)$ must be independent of the initial conditions. Such a parametric plot is shown in the top panel of fig. 1 for the most inelastic gas ($\alpha = 0.5$) and for the two heating cases ($a = 4/\tau^0$ and $a = 10/\tau^0$). The curve representing the rheological model (8) is also included. It is quite apparent that the ten curves are attracted to a common universal curve which, in addition, turns out to be excellently described by our simplified model. The bottom panel of fig. 1 displays the temporal evolution of $a^*(t)$, as measured by the accumulated number of collisions per particle (*i.e.*, the total number of collisions in the simulations divided by the total number of particles). We can observe that the states with the highest heating effect ($a = 10/\tau^0$) reach the hydrodynamic stage after about 1 collision per particle only, while this aging period takes a little longer (about 2 collisions per particle) in the states with $a = 4/\tau^0$. Once the state joins the hydrodynamic curve, it moves along it until the steady state is reached, what occurs after typically 10 collisions per particle since the initial state. This means that about 80–90% of the duration (measured by the number of collisions) of the evolution to the steady state is occupied by the hydrodynamic stage. It is also interesting to remark that, for a given value of a , the slowest heating takes place for the initial condition B0, followed

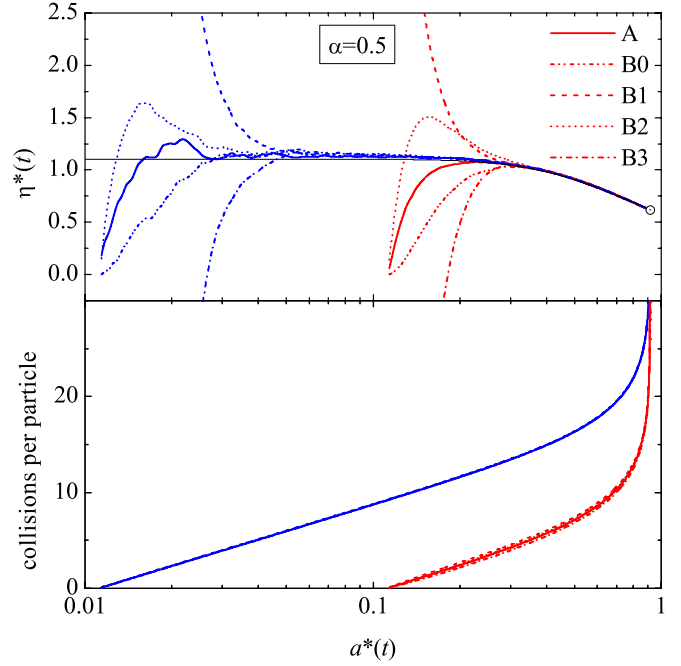


Fig. 2: (Color online) Same as in fig. 1, but for the two cooling cases $a = 0.01/\tau^0$ (blue lines) and $a = 0.1/\tau^0$ (red lines). Note the logarithmic scale in the horizontal axes and the linear scale in the vertical axis of the bottom panel.

by B3, A and B1, while the fastest heating corresponds to B2. This implies that the state starting from B0 is the one joining the hydrodynamic curve at a larger value of a^* ($a^* \simeq 7$ and $a^* \simeq 3$ for $a = 10/\tau^0$ and $a = 4/\tau^0$, respectively), while the one starting from B2 does it at a smaller value ($a^* \simeq 3$ and $a^* \simeq 1.5$ for $a = 10/\tau^0$ and $a = 4/\tau^0$, respectively).

Let us consider now the cases $a = 0.01/\tau^0$ and $a = 0.1/\tau^0$. The temporal evolution is dominated now by the inelastic cooling (*i.e.*, the time scale ζ^{-1} associated with the cooling is shorter than the time scale $\frac{3}{2}nT/a|P_{xy}|$ associated with the viscous heating) and so the existence of a hydrodynamic regime might appear as more doubtful than in the heating cases. That this is not actually the case can be concluded from fig. 2. Again, the ten curves tend to collapse to a common curve and again the latter practically coincides with the theoretical prediction (8), except perhaps near the NS region of small a^* . In contrast to the heating cases depicted in fig. 1, however, the temporal evolution of $T(t)$, and hence of $a^*(t) \propto T^{-1/2}(t)$, is practically independent of the initial condition, especially in the case $a = 0.01/\tau^0$. This is a consequence of the fact that for these low values of $a\tau^0$ the viscous term in eq. (1) can be neglected for short times and so the temperature initially evolves as in the homogeneous cooling state (decaying practically exponentially with the number of collisions), hardly affected by the details of the initial state. By the time the viscous heating effect becomes comparable to the inelastic cooling, the hydrodynamic regime

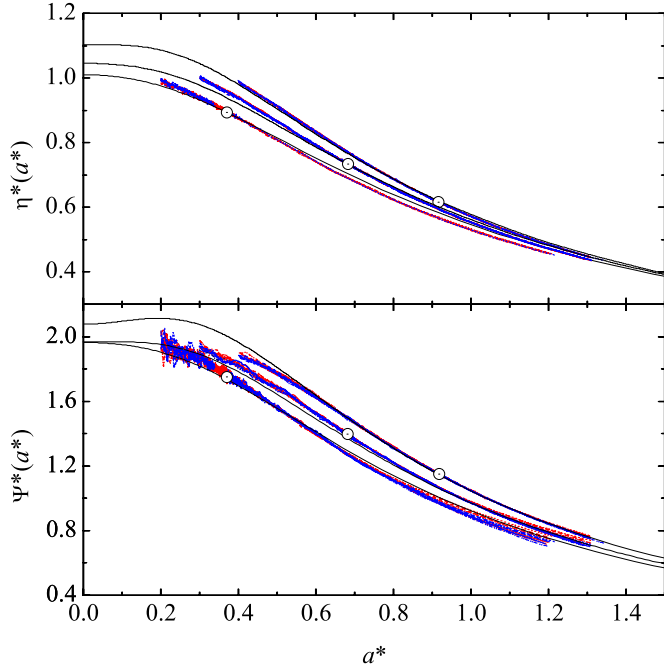


Fig. 3: (Color online) Reduced shear viscosity $\eta^*(a^*)$ (top panel) and viscometric function $\Psi^*(a^*)$ (bottom panel) for the hydrodynamic part of the evolution to the steady state (represented by a circle) for, from top to bottom, $\alpha = 0.5, 0.7$ and 0.9 . The color code is the same as in figs. 1 and 2. The thin solid lines correspond to the hydrodynamic functions, eqs. (8) and (9), obtained from our simplified rheological model.

has already been attained. It is important to note that the number of collisions needed to reach the hydrodynamic behavior (about 5 and 10 collisions per particle for $a = 0.01/\tau^0$ and $a = 0.1/\tau^0$, respectively) is longer in the cooling cases than in the heating cases. On the other hand, the total duration of the evolution period is also longer than in the heating cases and so the granular gas still lies on the hydrodynamic curve for most of the evolution time (as measured by the number of collisions per particle). The duration of the transient kinetic stage in the cooling cases (about 5–10 collisions per particle) is consistent with the one observed in molecular dynamics simulations of the homogeneous cooling state [14].

We have observed behaviors similar to those of figs. 1 and 2 for the viscometric function and for the other inelasticities ($\alpha = 0.7$ and $\alpha = 0.9$). While the number of collisions the system needs to loss memory of its initial state is practically independent of α , the total duration of the evolution period tends to increase with α , especially in the cooling cases [22]. This means that the more elastic the system, the smaller the fraction of time spent in the kinetic regime prior to the hydrodynamic stage.

The common hydrodynamic parts of the functions $\eta^*(a^*)$ and $\Psi^*(a^*)$ for the ten heating cases and the ten cooling cases are displayed in fig. 3 for each one of the three inelasticities considered. This corresponds to the

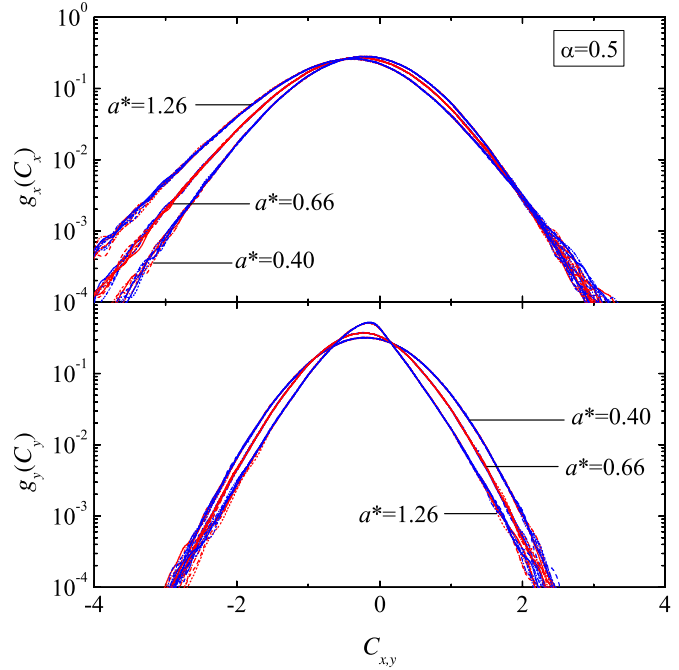


Fig. 4: (Color online) Marginal distribution functions $g_x(C_x)$ and $g_y(C_y)$ for $\alpha = 0.5$ and $a^* = 0.40, 0.66$ and 1.26 . The color code is the same as in figs. 1 and 2.

shear rate windows $0.4 \leq a^* \leq 1.3$, $0.3 \leq a^* \leq 1.3$ and $0.2 \leq a^* \leq 1.2$ for $\alpha = 0.5, 0.7$ and 0.9 , respectively. The degree of overlapping of the curves is excellent, although the statistical fluctuations inherent to the DSMC method are more important for $\Psi^*(a^*)$ than for $\eta^*(a^*)$, especially in the cooling branch of the curves. It is also apparent that our simplified model, eqs. (8) and (9), describes reasonably well the rheological properties of the sheared granular gas

Figures 1–3 confirm eq. (4), *i.e.*, the existence of well defined hydrodynamic rheological functions $P_{ij}^*(a^*)$ acting as attractors in the evolution of the pressure tensor $P_{ij}(t|f^0)$, regardless of the initial preparation f^0 . This in turn provides indirect support to the stronger statement (3). To test it in a more direct way, we have considered the marginal distributions [24] $g_x(C_x; a^*)$ and $g_y(C_y; a^*)$, defined by

$$g_{x,y}(C_{x,y}; a^*) = \int_{-\infty}^{\infty} dC_z \int_0^{\infty} dC_{y,x} f^*(\mathbf{C}; a^*), \quad (12)$$

for $\alpha = 0.5$ and three values of a^* that, according to figs. 1–3, are within the hydrodynamic range, namely $a^* = 0.40, 0.66$ (cooling branch) and 1.26 (heating branch). The results are plotted in fig. 4, which shows an excellent overlapping of the ten curves for each value of a^* , although obviously the tails of the distribution exhibit larger statistical fluctuations than the thermal region. As expected, the anisotropic features of the velocity distribution increase with the shear rate. The steady state corresponds to $a_s^* = 0.92$ [24] and hence the shapes of the associated steady distributions (not shown) lie in between those for

$a^* = 0.66$ and $a^* = 1.26$.

In conclusion, the results reported in this paper, along with a more extensive study that will be published elsewhere [22], strongly support that the conventional scenario of aging to hydrodynamics remains essentially valid for granular gases, even at high dissipation, for non-Newtonian states and for situations where the time scale associated with inelastic cooling is shorter than the one associated with the irreversible fluxes.

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